Zone plate

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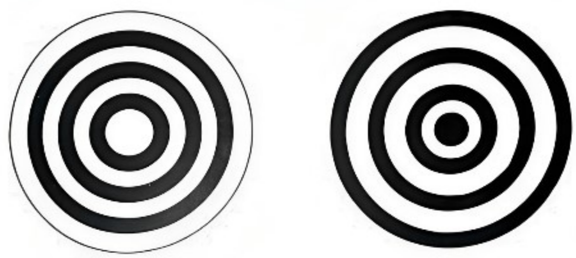
Zone plate :-

A zone is spherically constructed screen such that light is obstructed from every alternate zone. The correctness of Fresnel's method in dividing a wavefront into half period zones can be verified with the help of zone plate.

A zone plate is constructed by drawing concentric circles on a white paper such that radii are proportional to the square root of the natural numbers. The odd numbered zones (i.e. $1^{st}, 3^{rd}, 5^{th}, \ldots$) are covered with black ink and reduced Photograph is taken.

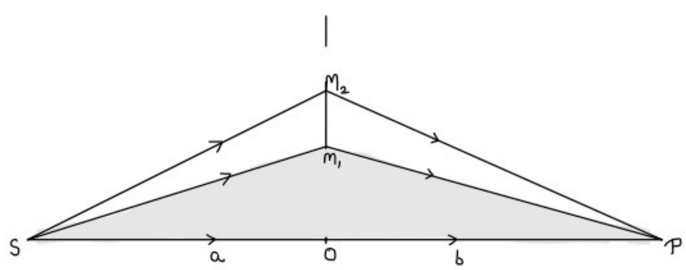


(a) Positive Zone Plate (b) Negative Zone Plate

The negative photograph appears is as shown in fig-(a). The negative shows odd zones are transparent to the incident light and even zone will cut off light. This is positive zone plate. If odd zones are opaque and the even zones are transparent then it is a negative zone plate. (Fig-(b))

THEORY :-

Let S be a point source of light of wavelength λ placed at a distance a from centre O of the zone plate. Let P be the point on a screen placed at distance b at which intensity of diffracted light bright.



Let $r_1, r_2, r_3, \ldots r_n$ be the radii of the $1^{st}, 2^{nd}, 3^{rd} \ldots n^{th}$ half period zones respectively. The position of the screen is such, that from one zone to the next there is an increasing path difference of $\frac{1}{2}$.

Thus from the diagram; $SO + OP = a + b$

$$SM_1 + M_1P = a + b + \frac{\lambda}{2} \qquad \text{——— (1)}$$

Similarly;

$$SM_2 + M_2P = a + b + \frac{2\lambda}{2} \quad \text{and so on}$$

From triangle SM_1O

$$SM_1 = \sqrt{(SO)^2 + (OM_1)^2} = \sqrt{a^2 + r_1^2}$$

Similarly, from the triangle PM_1O

$$M_1P = \sqrt{(OP)^2 + (OM_1)^2} = \sqrt{b^2 + r_1^2}$$

Substituting the values of SM_1 and M_1P in eq^n (1), we get :

$$\sqrt{a^2 + r_1^2} + \sqrt{b^2 + r_1^2} = a + b + \frac{\lambda}{2}$$

or

$$a\sqrt{1 + \frac{r_1^2}{a^2}} + b\sqrt{1 + \frac{r_1^2}{b^2}} = a + b + \frac{\lambda}{2}$$

Expanding and simplifying the above eq^n, we get :

$$a\left[1 + \frac{r_1^2}{2a^2}\right] + b\left[1 + \frac{r_1^2}{2b^2}\right] = a + b + \frac{\lambda}{2}$$

$$a + \frac{r_1^2}{2a} + b + \frac{r_1^2}{2b} = a + b + \frac{\lambda}{2}$$

or

$$\frac{r_1^2}{2}\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{\lambda}{2}$$

$$r_1^2\left[\frac{1}{a} + \frac{1}{b}\right] = \lambda$$

Thus for the radius of the n^{th} zone, the above relation can be written as :-

$$r_n^2\left[\frac{1}{a} + \frac{1}{b}\right] = n\lambda \qquad \text{——— (2)}$$

$$r_n^2 = \frac{ab}{a+b}\, n\lambda$$

$$r_n = \sqrt{\frac{ab\lambda}{a+b}}\,\sqrt{n} \qquad \Rightarrow \qquad \boxed{r_n \propto \sqrt{n}}$$

Thus, the radii of the half period zones are proportional to the square root of natural numbers.

From eq^n (2) can written as :-

$$\left[\frac{1}{a} + \frac{1}{b}\right] = \frac{n\lambda}{r_n^2} \qquad \text{—— (3)}$$

This equation is similar to the lens formula :

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad \text{—— (4)}$$

Comparing equation (3) and (4)

$$\boxed{\frac{1}{f} = \frac{n\lambda}{r_n^2}}$$

$$\boxed{f = \frac{r_n^2}{n\lambda}}$$

f is the focal length of zone plate and acts as a convex lens of multiple foci.

The path difference between any successive transparent zones is λ and the phase difference 2π. Waves from successive zones reach P in phase.